Final Exam

Instructions

- There are 6 questions worth a total of 54 points. 100%=50 points.
- No notes or books. A table of integration formulas is provided.
- You ***may*** use a simple scientific calculator. ***No*** graphing or programmable calculators.
- ***Take your time. Answer each question completely. Check your answers.***
- ***For full credit—explain/show your work.***

Good Luck!!!

NAME: ______________________________

Problem	Score
1	/9
2	/9
3	/9
4	/9
5	/9
6	/9
Total	/54

Selected Integration Formulas

Basic rules.

1. $\displaystyle\int u^k\,du = \frac{u^{k+1}}{k+1} + C, \quad k \neq -1.$

2. $\displaystyle\int \frac{1}{u}\,du = \ln|u| + C.$

3. $\displaystyle\int e^u\,du = e^u + C.$

4. $\displaystyle\int f(u) \pm g(u)\,du = \int f(u)\,du \pm \int g(u)\,du.$

5. $\displaystyle\int c\cdot f(u)\,du = c\cdot\int f(u)\,du.$

Rational forms containing $(\mathbf{a}+\mathbf{bu})$*.*

6. $\displaystyle\int \frac{du}{a+bu} = \frac{1}{b}\ln|a+bu| + C.$

7. $\displaystyle\int \frac{u\,du}{a+bu} = \frac{u}{b} - \frac{a}{b^2}\ln|a+bu| + C.$

8. $\displaystyle\int \frac{u^2\,du}{a+bu} = \frac{u^2}{2b} - \frac{au}{b^2} + \frac{a^2}{b^3}\ln|a+bu| + C.$

9. $\displaystyle\int \frac{u^2\,du}{(a+bu)^2} = \frac{u}{b^2} - \frac{a^2}{b^3(a+bu)} - \frac{2a}{b^3}\ln|a+bu| + C.$

Forms containing $\sqrt{\mathbf{a}+\mathbf{bu}}$*.*

10. $\displaystyle\int u\sqrt{a+bu}\,du = \frac{2(3bu-2a)(a+bu)^{3/2}}{15b^2} + C.$

11. $\displaystyle\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2(bu-2a)\sqrt{a+bu}}{3b^2} + C.$

12. $\displaystyle\int \frac{u^2\,du}{\sqrt{a+bu}} = \frac{2(3b^2u^2-4abu+8a^2)\sqrt{a+bu}}{15b^3} + C.$

Exponential and logarithmic forms.

13. $\displaystyle\int e^{au}\,du = \frac{e^{au}}{a} + C.$

14. $\displaystyle\int ue^{au}\,du = \frac{e^{au}}{a^2}(au-1) + C.$

15. $\displaystyle\int u^n e^{au}\,du = \frac{u^n e^{au}}{a} - \frac{n}{a}\int u^{n-1}e^{au}\,du\,.$

16. $\displaystyle\int u^n \ln u\,du = \frac{u^{n+1}\ln u}{n+1} - \frac{u^{n+1}}{(n+1)^2} + C, \qquad n \neq -1.$

1. (9 pts) A firm's marginal cost function is given by

$$\frac{dc}{dq} = \frac{8q}{\sqrt{2q+15}}.$$

Calculate the *total change* in the firm's cost if they increase their output from $q_1 = 5$ to $q_2 = 17$.

2. The demand function for a monopolist firm's product is given by $q = f(p)$, where q is daily demand and p is the price the firm sets. The price elasticity of demand for the firm's product is proportional to $\sqrt{p}$. When the price is $p_1 = \$9$, demand for the good is $q_1 = 400$ units/day. When the price is $p_2 = \$16$, the demand for the good is $q_2 = 300$ units/day.

(a) (3 pts) Write down a differential equation that is satisfied by the demand function for the firm's product.

(b) (6 pts) Find the demand function. What will be the firm's daily ***revenue*** if the price it sets is $p_3 = \$25$?

3. A household's utility function is given by

$$U(x, y, z) = 8 \ln x + 10 \ln y + 7 \ln z,$$

where x, y and z are the quantities of *Xidgets, Yidgets* and *Zidgets*, respectively, consumed by the household each month. The prices per unit for these three goods are $p_x = \$5$, $p_y = \$15$ and $p_z = \$20$, respectively.

(a) (6 pts) Find the quantities of Xidgets, Yidgets and Zidgets that should be consumed each month to maximize the household's utility, given that their monthly disposable income is $Y_d = \$3750$.

(b) (3 pts) By approximately how much will the household have to increase their monthly disposable income (from its current level) to increase their (maximum) utility by 3 utils? Explain your answer briefly.

4. The production function for ACME WIDGETS is given by

$$Q = 10K^{0.6}L^{0.3},$$

where Q is the number of widgets ACME produces in one year, K is the number of units of capital input and L is the number of units of labor input ACME uses to produce their widgets.

The price per unit of capital input is $p_K = \$2,000$ and the price per unit of labor input is $p_L = \$1000$.

(a) (6 pts) Find the levels capital and labor input that ***minimize the cost*** of producing $5,000$ widgets. What is the corresponding minimum cost?

(b) (3 pts) By approximately how much will the firm's minimum cost increase, if they increase their output by $\Delta Q = 50$ widgets? Show your work.

5. The monthly demand Q for a firm's product in a certain market is related to the price of their product, P, and the average monthly household income in the market, Y, by the equation:

$$Q = 50 \ln\left(5Y - 2P^3\right).$$

(a) (6 pts) Compute Q, Q_P, and Q_Y when $P = 20$ and $Y = 3500$.

(b) (2 pts) Compute the *income-elasticity of demand* at the same point as in (a).

(c) (1 pts) Use your answer to (b) to estimate the ***percentage change*** in demand if the average income increases by 2% from its value in (a).

6. A monopolistic firm sells one product in two markets, A and B. The daily demand equation for the firm's product in each market are given by

$$Q_A = 100 - 0.4P_A \qquad \text{and} \qquad Q_B = 120 - 0.5P_B,$$

where Q_A and Q_B are the demands and P_A and P_B are the prices for the firm's product in these markets. The firm's constant marginal cost is \$40 and the its daily fixed cost is \$2500.

(a) (7 pts) Find the prices that the firm should charge in each market to maximize its daily profit, and calculate the corresponding (maximum) profit. Use the second derivative test to verify that the prices you found yield the maximum profit.

(b) (2 pts) Use the *envelope theorem* (and linear approximation) to estimate the change in the firm's maximum daily profit, if the *marginal cost* increases from \$40 to \$42.