Final Exam

Instructions

- There are 6 questions worth a total of 54 points. 100%=50 points.

- No notes or books. A table of integration formulas is provided.

- You **may** use a simple scientific calculator. **No** graphing or programmable calculators.

- *Take your time. Answer each question completely. Check your answers.*

- *For full credit—explain/show your work.*

Good Luck!!!

NAME: _____

Problem	Score
1	/12
2	/6
3	/10
4	/8
5	/10
6	/8
Total	/60

Selected Integration Formulas

Basic rules.

1. $\displaystyle \int u^k \, du = \frac{u^{k+1}}{k+1} + C, \quad k \neq -1.$

2. $\displaystyle \int \frac{1}{u} \, du = \ln|u| + C.$

3. $\displaystyle \int e^u \, du = e^u + C.$

4. $\displaystyle \int f(u) \pm g(u) \, du = \int f(u) \, du \pm \int g(u) \, du.$

5. $\displaystyle \int c \cdot f(u) \, du = c \cdot \int f(u) \, du.$

Rational forms containing $(\mathbf{a + bu})$.

6. $\displaystyle \int \frac{du}{a + bu} = \frac{1}{b} \ln|a + bu| + C.$

7. $\displaystyle \int \frac{u \, du}{a + bu} = \frac{u}{b} - \frac{a}{b^2} \ln|a + bu| + C.$

8. $\displaystyle \int \frac{u^2 \, du}{a + bu} = \frac{u^2}{2b} - \frac{au}{b^2} + \frac{a^2}{b^3} \ln|a + bu| + C.$

9. $\displaystyle \int \frac{u^2 \, du}{(a + bu)^2} = \frac{u}{b^2} - \frac{a^2}{b^3(a + bu)} - \frac{2a}{b^3} \ln|a + bu| + C.$

Forms containing $\sqrt{\mathbf{a + bu}}$.

10. $\displaystyle \int u\sqrt{a + bu} \, du = \frac{2(3bu - 2a)(a + bu)^{3/2}}{15b^2} + C.$

11. $\displaystyle \int \frac{u \, du}{\sqrt{a + bu}} = \frac{2(bu - 2a)\sqrt{a + bu}}{3b^2} + C.$

12. $\displaystyle \int \frac{u^2 \, du}{\sqrt{a + bu}} = \frac{2(3b^2u^2 - 4abu + 8a^2)\sqrt{a + bu}}{15b^3} + C.$

Exponential and logarithmic forms.

13. $\displaystyle \int e^{au} \, du = \frac{e^{au}}{a} + C.$

14. $\displaystyle \int u e^{au} \, du = \frac{e^{au}}{a^2}(au - 1) + C.$

15. $\displaystyle \int u^n e^{au} \, du = \frac{u^n e^{au}}{a} - \frac{n}{a} \int u^{n-1} e^{au} \, du.$

16. $\displaystyle \int u^n \ln u \, du = \frac{u^{n+1} \ln u}{n+1} - \frac{u^{n+1}}{(n+1)^2} + C, \qquad n \neq -1.$

1. (a) (6 pts) Compute the *present value* of a continuous annuity that pays at the annual rate $f(t) = 1500t$ for $T = 10$ years, assuming that interest is compounded continuously at the rate $r = 3\%$.

(b) (6 pts) Compute the *Gini coefficient* (of inequality) for the nation whose income distribution function is given by $f(x) = 0.8x^2 + 0.2x$.

2. (6 pts) The *price elasticity of demand*, $\eta_{q/p}$, for a monopolistic firm's product is ***proportional to*** the square root of p, the price of the firm's good.[†] Find the demand function $q = h(p)$ for the firm's product, given that $h(9) = 200$ and $h(16) = 120$.

[†]I.e., $\eta_{q/p} = k\sqrt{p}$, where k is the constant of proportionality.

3. A household's utility function is given by

$$U(x, y, z) = 10 \ln x + 8 \ln y + 7 \ln z,$$

where x, y and z are the quantities of *Xidgets, Yidgets* and *Zidgets*, respectively, consumed by the household each month. The prices per unit for these three goods are $p_x = \$15$, $p_y = \$10$ and $p_z = \$5$, respectively.

(a) (6 pts) Find the quantities of Xidgets, Yidgets and Zidgets that should be consumed each month to maximize the household's utility, given that their monthly XYZ-budget is $B = \$6000$.

(b) (2 pts) By approximately how much will the household have to increase their monthly XYZ-budget from its current level to increase their (maximum) utility by 3 utils? Explain your answer briefly.

(c) (2 pts) By approximately how much will the household's (maximum) utility change from the value you found in **a.**, if the average price of a Yidget increases by $1, assuming that the other prices and their budget stay the same? Justify your answer in terms of the *envelope theorem*.

4. (8 pts) Find the critical points of the function

$$g(x, y) = x^2 + 4xy + 2y^2 + y^3 + 2x + 5$$

and classify the critical values using the second derivative test.

5. The average monthly demand (q) for a monopolistic firm's product is related to the price of their product (p), the average price of substitutes for their product (p_s) and the average monthly household income in the market for the firm's product (y), by the equation:

$$q = \frac{150(2y + 20p_s + 352)^{2/3}}{4p + 50}.$$

(a) (6 pts) Compute $\partial q/\partial p$, $\partial q/\partial p_s$, and $\partial q/\partial y$ when $p = 25$, $p_s = 24$ and $y = 2500$.

(b) (2 pts) Suppose that income stays fixed, but both prices increase by \$1. Use your answer to **a.** to estimate the change in demand for the firm's product.

(c) (2 pts) What is the income-elasticity of demand at the point in part **a.**?

6. The production function for ICME WADGETS is given by

$$Q = 20K^{0.7}L^{0.3},$$

where Q is the number of wadgets ICME produces in one year, K is the number of units of capital input and L is the number of units of labor input ICME uses to produce their wadgets.

The price per unit of capital input is $p_K = \$5,000$ and the price per unit of labor input is $p_L = \$3000$.

(a) (6 pts) Find the levels capital and labor input that ***minimize the cost*** of producing $q = 10,000$ wadgets. What is the corresponding minimum cost?

(b) (2 pts) How much would it cost ICME to produce another 50 wadgets? Is your answer precise or approximate? Explain.