1. (5 pts) $\displaystyle\int_1^5 \frac{3}{x(5+2x)}\,dx = \frac{3}{5}\ln\left|\frac{x}{5+2x}\right|\Bigg|_1^5 = \frac{3}{5}\left(\ln\frac{5}{15} - \ln\frac{1}{7}\right) = \frac{3}{5}\ln\frac{7}{3} \quad (\approx 0.508)$

Comment: Use the formula $\displaystyle\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$

2. (5 pts) Find the present value of a continuous income stream that pays at the annual rate $f(t) = 500t$ for $T = 20$ years, assuming that the interest rate is $r = 4.5\%$.

$$\begin{aligned}\text{Present Value} &= \int_0^T f(t)e^{-rt}\,dt\\ &= \int_0^{20} 500te^{-0.045t}\,dt\\ &= 500\,\frac{e^{-0.045t}}{(-0.045)^2}(-0.045t-1)\Bigg|_0^{20} = \frac{500}{(0.045)^2}\left(e^{-0.9}(-1.9)+1\right) \approx 56177.2\end{aligned}$$

3. (5 pts) The price-elasticity of demand for a certain good is assumed to be proportional to the square-root of its price. When the price is $p_0 = 9$, the demand is $q_0 = 540$ and when the price is $p_1 = 16$, the demand is $q_1 = 405$. What will the demand be if the price increases to $p = 25$?

Differential equation: $\eta_{q/p} = \dfrac{dq}{dp}\cdot\dfrac{p}{q} = kp^{1/2}$.

Comment: k is the (unknown) constant of proprtionality.

Separate: $\dfrac{1}{q}\,dq = kp^{-1/2}\,dp$

Integrate: $\displaystyle\int \frac{1}{q}\,dq = \int kp^{-1/2}\,dp \implies \ln|q| = 2kp^{1/2} + C \implies \ln q = kp^{1/2} + C$

Comments: The (unknown) constant k can absorb the factor of 2 (if you choose). Also, we are assuming that $q > 0$ because it represents demand, so $\ln|q| = \ln q$.

Solve for q (exponentiate): $q = e^{kp^{1/2}+C} = e^C \cdot e^{kp^{1/2}} = Ae^{kp^{1/2}} \qquad (A = e^C)$

Use data to solve for A and k: First...

$$\left.\begin{aligned} 540 &= q(9) &= Ae^{k\sqrt{9}}\\ 405 &= q(16) &= Ae^{k\sqrt{16}}\end{aligned}\right\} \implies \frac{\cancel{A}e^{3k}}{\cancel{A}e^{4k}} = \frac{540}{405} \implies e^{-k} = \frac{4}{3} \implies -k = \ln(4/3) \implies k = -\ln(4/3)$$

Next...

$$540 = A\cdot e^{-3\ln(4/3)} = \frac{1}{(4/3)^3} = A\cdot(3/4)^3 = \frac{27A}{64} \implies A = \frac{64\cdot 540}{27} = 1280$$

Finally

$$q(25) = 1280e^{-\ln(4/3)\cdot\sqrt{25}} = 1280\cdot(3/4)^5 = 303.75$$

Bonus (2 pts) Find the indicated partial derivatives of the function $f(x,y) = 3x^2\ln(2x+5y)$. Clean up your answer.

$\dfrac{\partial f}{\partial x} = 6x\ln(2x+5y) + 3x^2\cdot\dfrac{1}{2x+5y}\cdot 2 = 6x\ln(2x+5y) + \dfrac{6x^2}{2x+5y}$ (product rule and chain rule)

$\dfrac{\partial f}{\partial y} = 3x^2\cdot\dfrac{1}{2x+5y}\cdot 5 = \dfrac{15x^2}{2x+5y}$ (no product rule for this one, because the factor $3x^2$ acts as a constant)