1. The demand function for a monopolistic firm's product is given by $q = 50y\ln(3y^2 - 2p)$, where q is the monthly demand for the firm's product, y is the average monthly income in the market for the firm's product, *measured in* \$1000*s*, and p is the price of the firm's product, *measured in dollars.*

(a) (5 pts) Find q, q_p and q_y when average income is \$4000 and the price is \$10.

Remember that y is measured in \$1000s, so an average income of \$4000 means that $y = 4$.
On to the computations:

$$q\Big|_{\substack{p=10\\y=4}} = 200\ln(48-20) = 200\ln 28 \approx 666.44$$

$$\frac{\partial q}{\partial p}\Big|_{\substack{p=10\\y=4}} = \frac{-100y}{3y^2-2p}\Big|_{\substack{p=10\\y=4}} = -\frac{400}{28} \approx -14.286$$

and

$$\frac{\partial q}{\partial y}\Big|_{\substack{p=10\\y=4}} = \left(50\ln(3y^2-2p) + \frac{300y^2}{3y^2-2p}\right)\Big|_{\substack{p=10\\y=4}} = 50\ln 28 + \frac{4800}{28} \approx 338.04$$

(b) (2 pts) Use linear approximation to estimate the change in demand if average income rises to \$4200 and the firm raises their price to \$10.50.

Once again remembering the units, we have $\Delta y = \frac{200}{4000} = 0.2$ and $\Delta p = 0.5$. Using Linear approximation, we obtain the estimate:

$$\Delta q \approx \frac{\partial q}{\partial p}\Big|_{\substack{p=10\\y=4}} \cdot \Delta p + \frac{\partial q}{\partial y}\Big|_{\substack{p=10\\y=4}} \Delta y = -\frac{400}{28}\cdot 0.5 + \left(50\ln 28 + \frac{4800}{28}\right)\cdot 0.2 \approx 60.465$$

2. (5 pts) Find the critical points and critical values of the function $w = 2s^3 - 3t^2 + 6st + 1$.

First order conditions (and solutions):

$$\begin{array}{lllllllll}
w_s = 0 & \Longrightarrow & 6s^2 + 6t = 0 & \Longrightarrow & 6s^2 + 6s = 0 & \Longrightarrow & 6s(s+1) = 0 & \Longrightarrow & s = 0 \text{ or } s = -1 \\
 & & & & \quad\uparrow & & & & \\
w_t = 0 & \Longrightarrow & -6t + 6s = 0 & \Longrightarrow & \overbrace{6t = 6s} \quad \Longrightarrow & \Longrightarrow & & \Longrightarrow & \underbrace{t = s} \\
 & & & & & & & & \quad\downarrow
\end{array}$$

I.e., the critical points are $(s_1, t_1) = (0, 0)$ and $(s_2, t_2) = (-1, -1)$ (since $t = s$ at the critical points) and the critical values are

$$w_1 = w(0,0) = 1 \ \text{ and } \ w_2 = w(-1,-1) = 2.$$

3. (3 pts) Use the second derivative test to classify the critical values that you found above as relative minima, relative maxima or neither.

Second derivatives: $w_{ss} = 12s$, $w_{tt} = -6$ and $w_{st} = 6$.
Discriminant: $D(s,t) = w_{ss}w_{tt} - w_{st}^2 = -72s - 36$.
Test:
$D(0,0) = -36 < 0$, so $w_1 = 1$ is neither min nor max (the point $(0,0,1)$ is a saddle point on the graph $w = 2s^3 - 3t^2 + 6st + 1$).
$D(-1,-1) = 36 > 0$ and $w_{ss}(-1,-1) = -12 < 0$, so $w_2 = 2$ is a relative max value.