

## The definite integral and Applications

1. Compute the definite integrals below.

a.  $\int_0^2 2x^3 + x^2 - 5x + 2 \, dx =$

d.  $\int_0^{20} 500e^{-0.04t} \, dt =$

b.  $\int_1^8 2\sqrt[3]{t} + \frac{3}{\sqrt[3]{t^2}} \, dt =$

e.  $\int_0^4 3t\sqrt{t^2 + 9} \, dt$

c.  $\int_0^4 \frac{5}{4x + 1} \, dx =$

2. Find the area of the region bounded by the graphs  $y = 2\sqrt{x}$  and  $y = 1 - 2x$ , and the lines  $x = 1$  and  $x = 4$ .

3. What are the *Producers' surplus* and *Consumers' surplus* for the market with supply function

$$p = 0.05q^2 + 3q + 5$$

and demand function

$$p = 100 - 0.75q.$$

4. Find the average value of the function  $f(x) = \frac{x^4 - 1}{x^2}$  on the interval  $[1, 3]$ .

5. Find the Gini coefficient of inequality for the nation with income distribution curve

$$y = 0.5x^3 + 0.3x^2 + 0.2x,$$

where  $y \cdot 100\%$  is the percentage of national income earned by the poorest  $x \cdot 100\%$  of the population.

6. The marginal propensity to consume of a small nation is given by

$$\frac{dS}{dY} = \frac{Y + 5}{9Y + 10},$$

where consumption  $S$  and national income  $Y$  are both measured in billions of dollars. Express the total change in national savings when income increases from \$10 billion to \$15 billion as a definite integral, and find its value. What is the total change in national consumption?