Study Guide 3

More applications of integration (and partial derivatives)

1. Use the table of integral formulas in Appendix B in the textbook to help compute the integrals below.

a. $\displaystyle\int \frac{4\,dx}{5x\sqrt{x^2+9}} =$

b. $\displaystyle\int \frac{2e^{2x}\,dx}{\sqrt{9+4e^x}} =$

c. $\displaystyle\int_0^{10} 200t^2e^{-0.06t}\,dt =$

d. $\displaystyle\int_0^3 \frac{2\,dv}{\sqrt{v^2+16}} =$

e. $\displaystyle\int 5x^3\ln x\,dx =$

f. $\displaystyle\int_0^2 \frac{3+5x}{2+7x}\,dx =$

2. Compute the present value of the continuous annuity that pays at the continuous rate $f(t) = 250t$ for $T = 20$ years, where the constant interest rate is $r = 4.75\%$.

3. Let $y = f(x)$ satisfy (i) $\dfrac{dy}{dx} = 3xy^2$ and (ii) $y(1) = 2$. Find the function $f(x)$.

4. The dead body of an eccentric socialite is found in a Las Vegas motel room. At 10:00 am, her body temperature was measured to be 92.4°F. Her body was left in the room for an hour and at 11:00 am her body temperature was 90.5°F. The room itself was kept at a constant temperature of 72°F.

Use *Newton's law of cooling* to estimate the time of the socialite's death.

(*) See Example 3 in section 15.6.

5. The population of a tropical island grows at a rate that is proportional to the *third root* ($\sqrt[3]{\ }$) of its size. In 1950, the islands population was 1728 and in 1980, the islands population was 2744. What will the islands population be in 2020?

6. The income-elasticity of monthly demand (q) for a price-controlled good is assumed to be proportional to the natural logarithm of average monthly disposable income (y) in the market for that good. When $y = 2500$, the demand is $q = 500$ and when $y = 2000$, the demand is $q = 350$. What is the predicted monthly demand for this good if monthly disposable income decreases to $y = 1500$?

7. Compute the indicated partial derivatives of the functions given below.

(a) $f(x,y,z) = 2x^3y^2z + 3x^2y^5z^3 - 4xy^3 + yz^4$. Find f_x, f_y and f_z

(b) $w = \dfrac{u^2v}{u+v^2}$. Find $\dfrac{\partial w}{\partial u}$ and $\dfrac{\partial w}{\partial v}$.

(c) $F(x,y,z,\lambda) = 10\ln(x^2y^5z^3) - \lambda(5x+2y+8z)$. Find F_x and F_λ.

(d) $z = 2y^2e^{x^2y}$. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.