Study Guide 4

Partial derivatives and applications; Optimization

1. Find the indicated partial derivatives of the functions below.

a. $z = 3x^2 + 4xy - 5y^2 - 4x + 7y - 2,$

$z_x =$

$z_{yx} =$

b. $F(u, v, w) = 60u^{2/3}v^{1/6}w^{1/2}$

$\dfrac{\partial F}{\partial u} =$

$\dfrac{\partial^2 F}{\partial w \partial u} =$

c. $w = x^2 z \ln(y^2 + z^3)$

$w_x =$

$w_y =$

$w_{xx} =$

$w_{yz} =$

$w_{xyz} =$

d. $q(u, v) = \dfrac{u^2 v - 3uv^3}{2u + 3v}$

$\dfrac{\partial q}{\partial u} =$

$\dfrac{\partial q}{\partial v} =$

2. The monthly cost function for ACME Widgets is

$$C = 0.02Q_A^2 + 0.01Q_A Q_B + 0.03Q_B^2 + 35Q_A + 28Q_B + 5000,$$

where Q_A and Q_B are the monthly outputs of type A widgets and type B widgets, respectively, measured in 100s of widgets. The cost is measured in dollars.

a. Compute the marginal cost of type A widgets and the marginal cost of type B widgets, if the monthly outputs are 25000 type A widgets and 36000 type B widgets.

b. Suppose that production of type A widgets is held fixed at 25000, and production of type B widgets is increased from 36000 to 36050. Use your answer to part a. to estimate the change in cost to the firm.

c. Suppose that production of type A widgets is increased from 25000 to 25060, and production of type B widgets is increased from 36000 to 36040. Use your answer to part a. to estimate the change in cost to the firm.

3. The demand function for a firm's product is given by $Q = \dfrac{30\sqrt{6Y + 5p_s}}{3p + 5}$, where

- Q is the monthly demand for the firm's product, measured in 1000's of units,
- Y is the average monthly disposable income in the market for the firm's product, measured in 1000s of dollars,
- p_s is the average price of a substitute for the firm's product, measured in dollars,
- p is the price of the firm's product, also measured in dollars.

a. Find Q, Q_Y, Q_{p_s} and Q_p when the monthly income is \$2500 and the prices are $p_s = 17$ and $p = 15$. Round your (final) answers to two decimal places.

b. Compute the *income-elasticity of demand* for the firm's product at the point in part **a.**

c. Use *linear approximation* and your answer to **a.** to estimate the change in demand for the firm's product if the price of the firm's product increases to \$16 and the price of substitutes increases to \$18, but income remains fixed.

d. Use your answer to part **b.** to estimate the *percentage* change in demand for the firm's product if the average income increases to \$2600 while the prices stay the same as they were in part a.

4. Find the critical points of the functions below.

a. $f(x, y) = 3x^2 - 12xy + 19y^2 - 2x - 4y + 5$.

b. $g(s, t) = s^3 + 3t^2 + 12st + 2$.

c. $h(u, v) = u^3 + v^3 - 3u^2 - 3v + 5$.

5. Use the second derivative test to classify the critical values of the functions in the previous problem.

6. ACME Widgets produces two competing products, type A widgets and type B widgets. The joint demand functions for these products are

$$Q_A = 100 - 3P_A + 2P_B \quad \text{and} \quad Q_B = 60 + 2P_A - 2P_B$$

and ACME's cost function is

$$C = 20Q_A + 30Q_B + 1200.$$

Find the prices that ACME should charge to maximize their profit, the corresponding output levels and the max profit. Justify your claim that the prices you found yield the absolute maximum profit.

7. An electronics retailer has determined that the number N of laptops she can sell per week is

$$N = \frac{9x}{4+x} + \frac{20y}{5+y}$$

where x is her weekly expenditure on radio advertising and y is her weekly expenditure on internet advertising, both measured in \$100s. Her weekly profit is \$400 per sale, less the cost of advertising.

Find the amount of money that the retailer should spend on radio and internet advertising, respectively, to maximize her weekly profit. Verify that the point you found yields a relative maximum value. What is the maximum profit?